O K L A H O M A   S T A T E   U N I V E R S I T Y

SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

ECEN 4503
Random Signals and Noise
Spring 2004

Midterm Exam #2

Choose any four out of five.
Please specify below which four you choose to be graded.

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Name : ________________________________

Student ID: ___________________________________

E-Mail Address: ____

Problem 1:

Determine constant a such that the function

$$F_{X,Y}(x,y) = a\left[\frac{\pi}{2} + \tan^{-1}\left(\frac{x}{2}\right)\right]\left[\frac{\pi}{2} + \tan^{-1}\left(\frac{y}{3}\right)\right]$$

is a valid joint distribution function.

Problem 2:

The probability density functions of two statistically independent random variables X and Y are

$$f_X(x) = \tfrac{1}{2}u(x-1)e^{-(x-1)/2}$$

$$f_Y(y) = \tfrac{1}{4}u(y-3)e^{-(y-3)/4}$$

Find the probability density function of the difference $W = X - Y$.

Problem 3:
The random variables X and Y are statistically independent with exponential densities

$f_X(x) = \alpha e^{-\alpha x} u(x)$, and

$f_Y(y) = \beta e^{-\beta y} u(y)$.

Find the probability density function of the random variable $W = \min(X, Y)$.

Problem 4:
Statistically independent random variables X and Y have moments $m_{10} = 2$, $m_{20} = 14$, $m_{02} = 12$, and $m_{11} = -6$. Find the moment μ_{22}.

<u>Problem 5</u>:

Two Gaussian random variables X_1 and X_2 are defined by the mean vector and covariance matrix of

$$\overline{X} = \begin{bmatrix} \overline{X}_1 \\ \overline{X}_2 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}, \qquad C_X = \begin{bmatrix} 5 & -2/\sqrt{5} \\ -2/\sqrt{5} & 4 \end{bmatrix}.$$

Two new random variables Y_1 and Y_2 are formed using the transformation

$$T = \begin{bmatrix} 1 & 1/2 \\ 1/2 & 1 \end{bmatrix}.$$

Find the mean vector of random variable Y, $\overline{Y}$, and covariance matrix of Y, C_Y. Also find the correlation coefficient of Y_1 and Y_2, ρ_{Y_1,Y_2}.